

Avertissement

La présentation qui suit ne comporte pas de commentaires. On trouvera des informations sur le même sujet dans les documents suivants :

http://people.math.jussieu.fr/~alp/topos_shadoks.pdf

http://people.math.jussieu.fr/~alp/tiers_exclu.pdf

Le lecteur intéressé par la logique catégorique peut consulter l'ouvrage très complet :

S. MacLane, I. Moerdijk **Sheaves in Geometry and Logic** Springer Universitext

ainsi que mon cours de logique catégorique téléchargeable à l'adresse :

<http://people.math.jussieu.fr/~alp>

Pourquoi y a-t-il une logique non classique ?

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12 mai 2009

Démonstration naturelle

$$\begin{array}{l} (=) \frac{}{\top \vdash_{\Gamma} a = a} \quad (\text{id}) \frac{}{E \vdash_{\Gamma} E} \quad (\circ) \frac{E \vdash_{\Gamma} F, F \vdash_{\Gamma} G}{E \vdash_{\Gamma} G} \end{array}$$

$$\begin{array}{l} (\text{dec}) \frac{E \vdash_{\Gamma} F}{E \vdash_{\Gamma(x \in X)} F} \quad (\text{rep}) \frac{E \vdash_{\Gamma(x \in X)} F}{E[c/x] \vdash_{\Gamma} F[c/x]} \end{array}$$

$$\begin{array}{l} (\top) \frac{}{E \vdash_{\Gamma} \top} \quad (\perp) \frac{}{\perp \vdash_{\Gamma} F} \end{array}$$

$$\begin{array}{l} (\wedge) \frac{H \vdash_{\Gamma} E, H \vdash_{\Gamma} F}{H \vdash_{\Gamma} E \wedge F} \quad (\vee) \frac{H \vdash_{\Gamma} F, K \vdash_{\Gamma} F}{H \vee K \vdash_{\Gamma} F} \end{array}$$

$$(\Rightarrow) \frac{H \wedge E \vdash_{\Gamma} F}{H \vdash_{\Gamma} E \Rightarrow F}$$

$$\begin{array}{l} (\forall) \frac{E \vdash_{\Gamma(x \in X)} F}{E \vdash_{\Gamma} \forall_{x \in X} F} \quad (\exists) \frac{H \vdash_{\Gamma(x \in X)} F}{\exists_{x \in X} H \vdash_{\Gamma} F} \end{array}$$

La négation

$$\neg E := E \Rightarrow \perp$$

$$(\neg) \frac{H \wedge E \vdash_{\Gamma} \perp}{H \vdash_{\Gamma} \neg E}$$

La co-négation

$$(\Rightarrow) \frac{H \wedge E \vdash_{\Gamma} F}{H \vdash_{\Gamma} E \Rightarrow F}$$

$$\neg E := E \Rightarrow \perp$$

$$(\neg) \frac{H \wedge E \vdash_{\Gamma} \perp}{H \vdash_{\Gamma} \neg E}$$

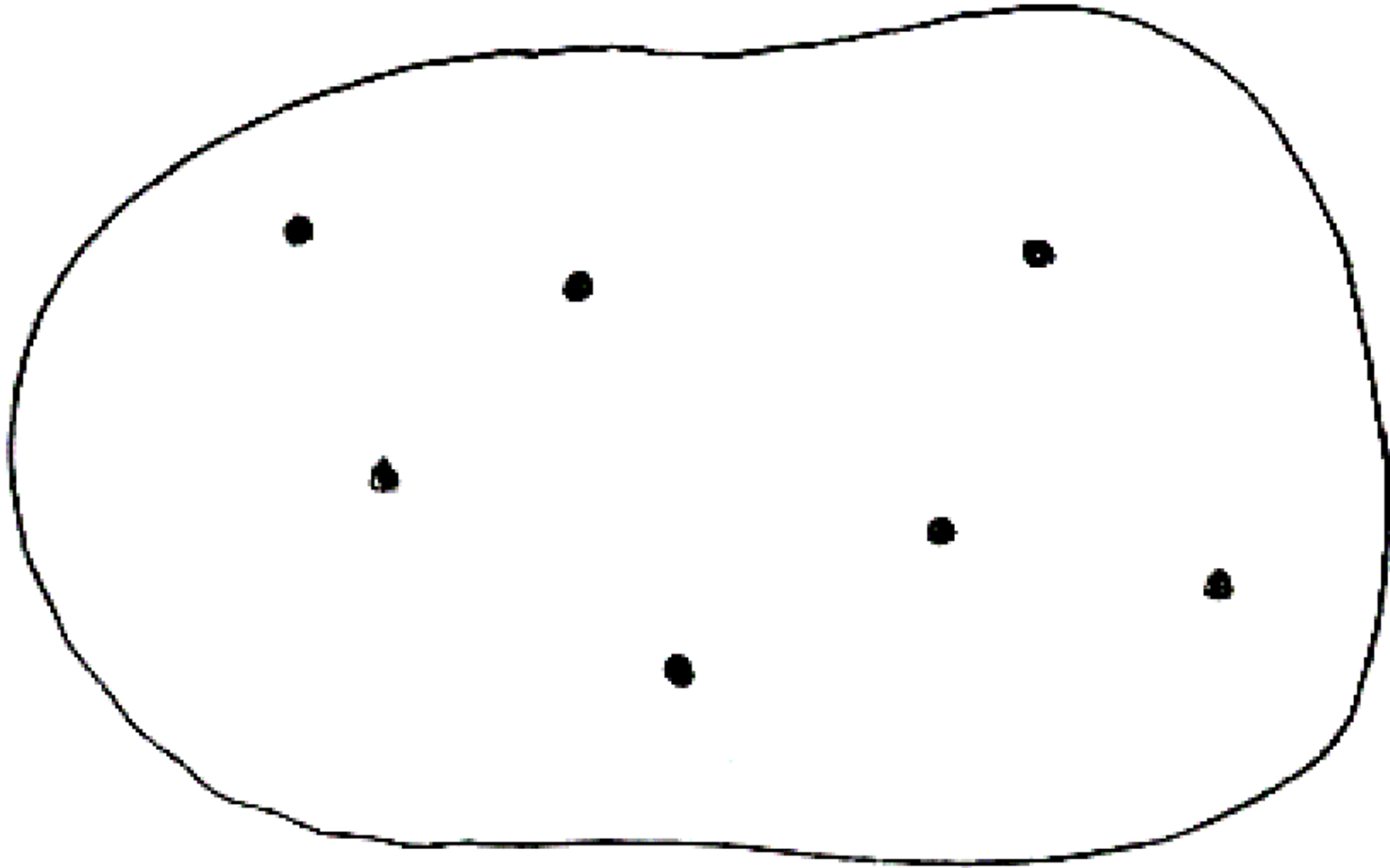
$$(\setminus) \frac{H \vdash_{\Gamma} E \vee F}{H \setminus E \vdash_{\Gamma} F}$$

$$\sim E := \top \setminus E$$

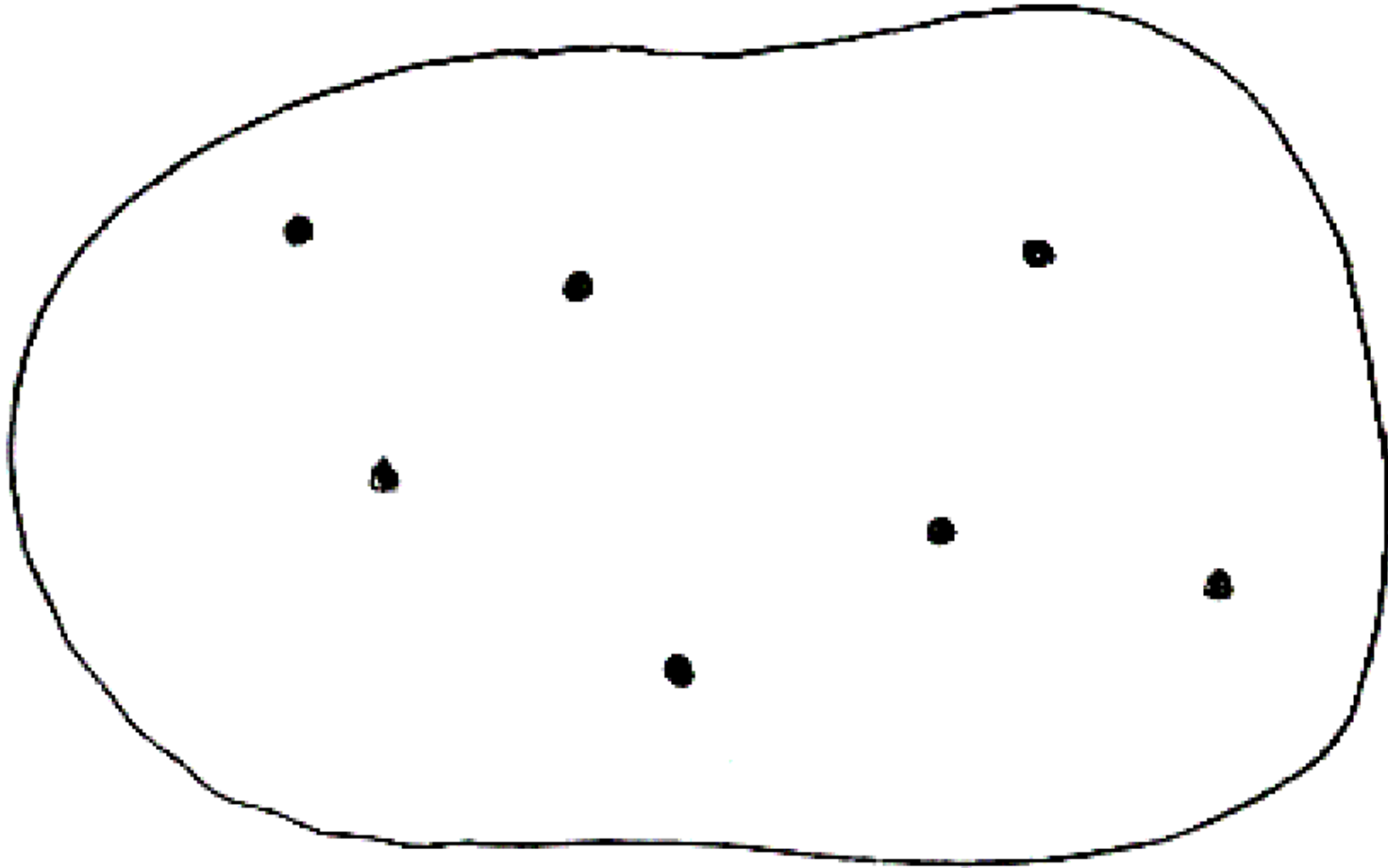
$$(\sim) \frac{\top \vdash_{\Gamma} E \vee F}{\sim E \vdash_{\Gamma} F}$$

symétrie/asymétrie

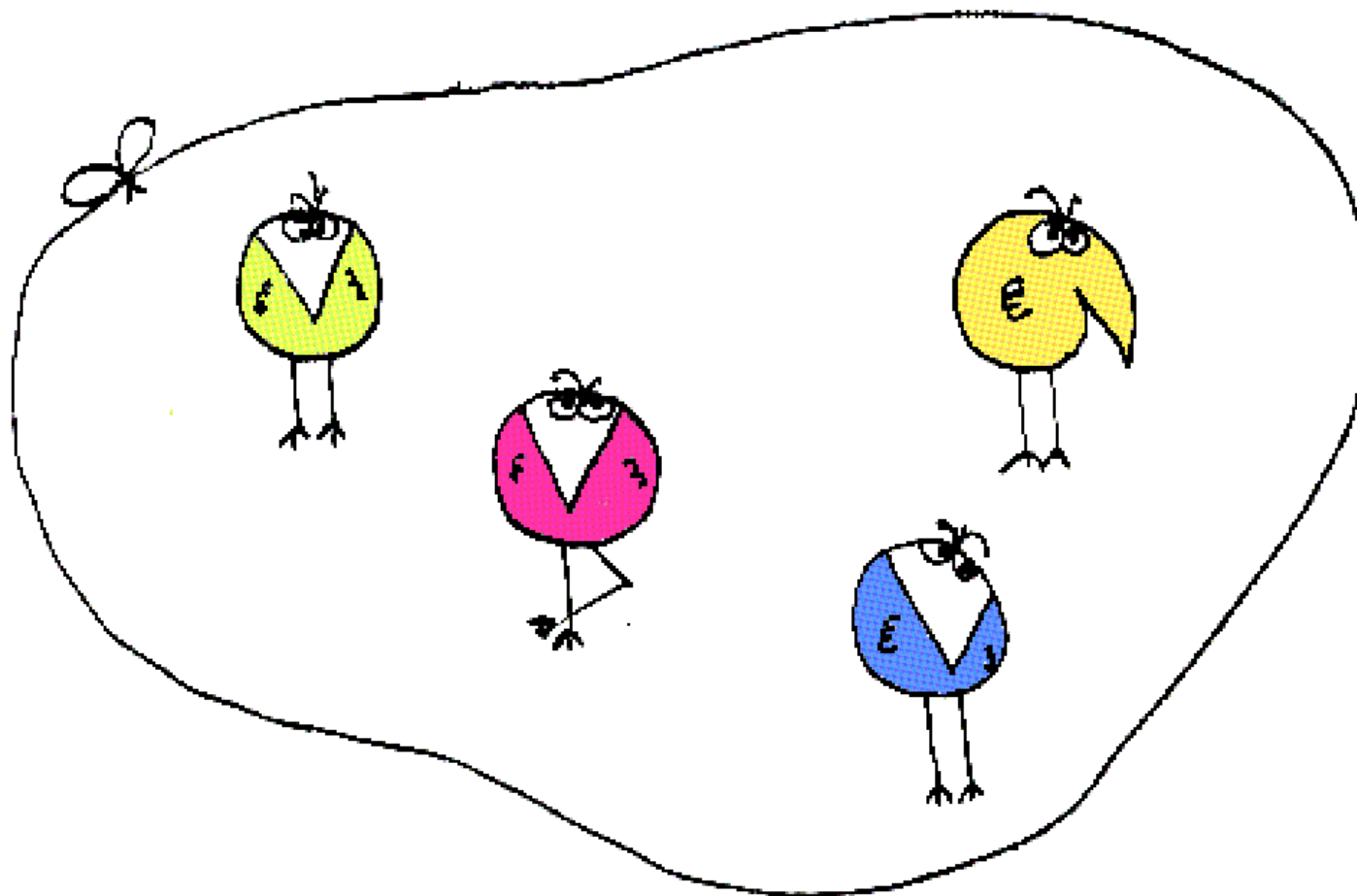
?



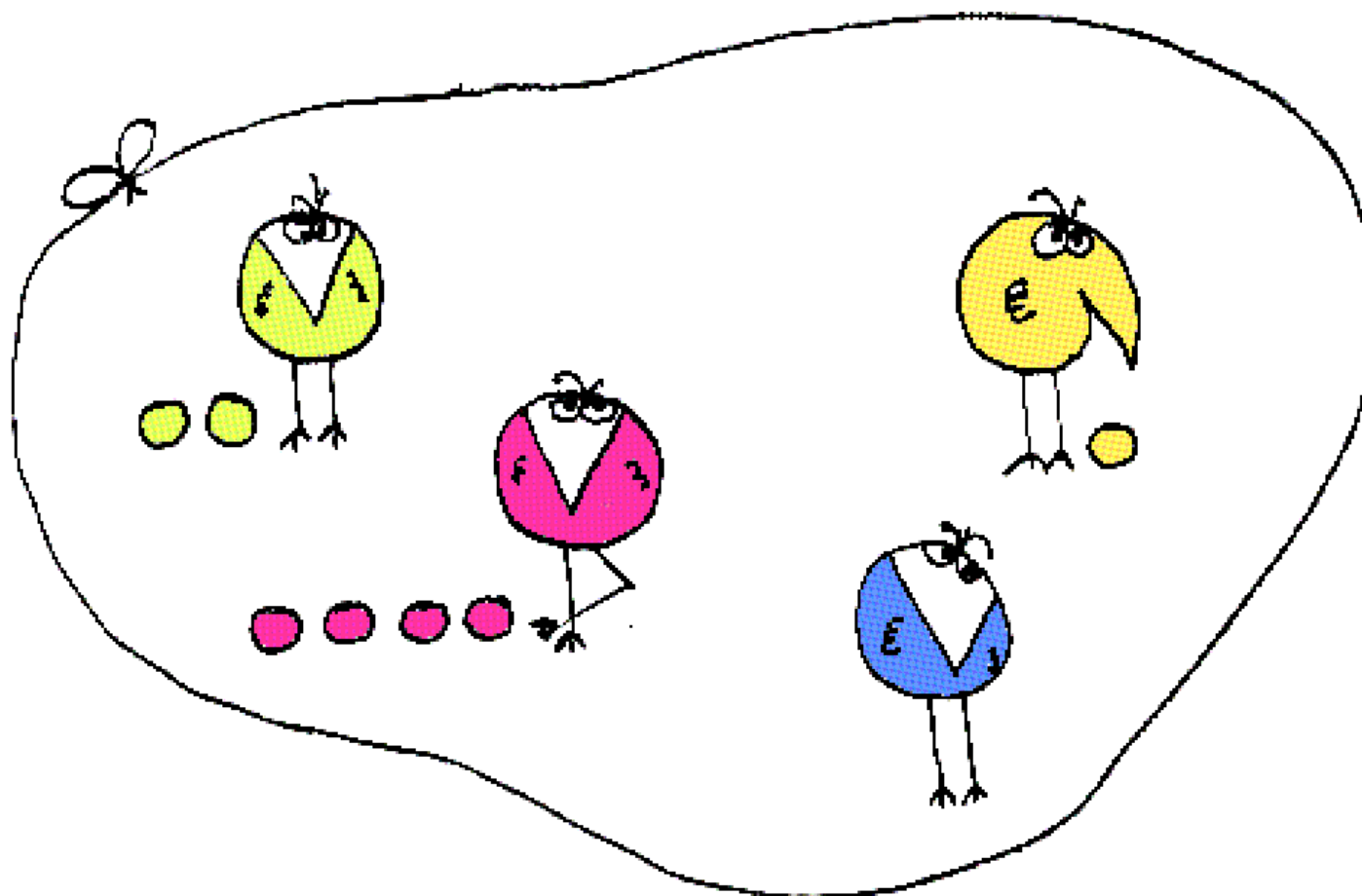
Ensemble



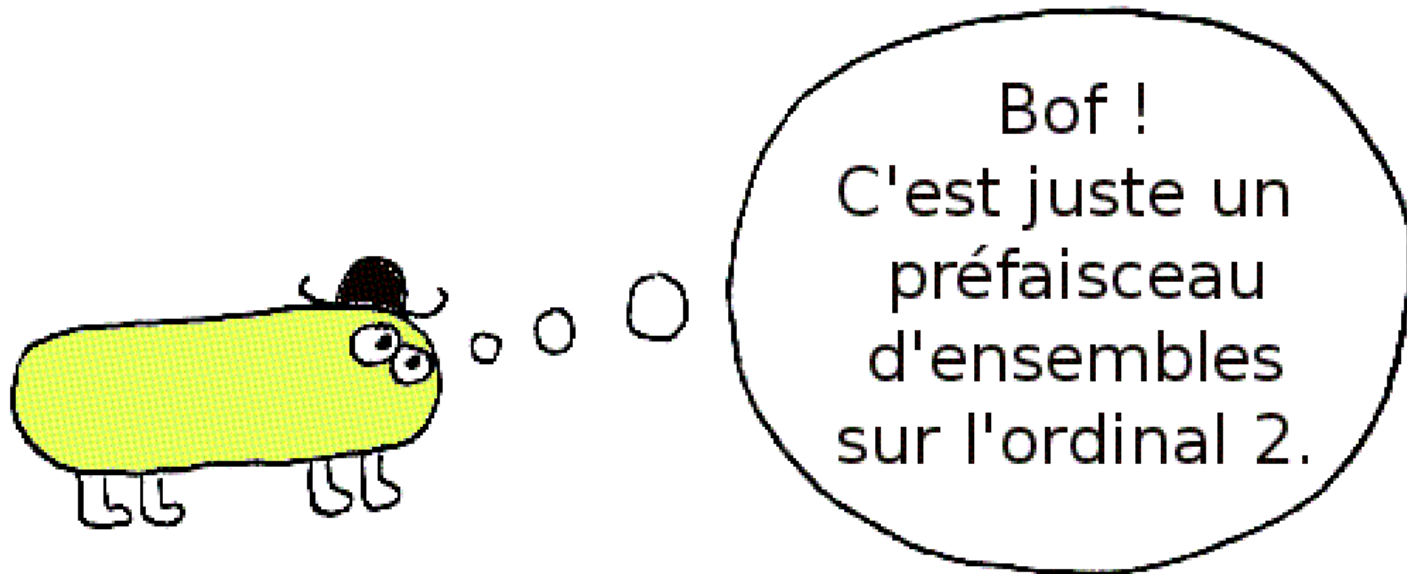
Ensemble de Shadoks



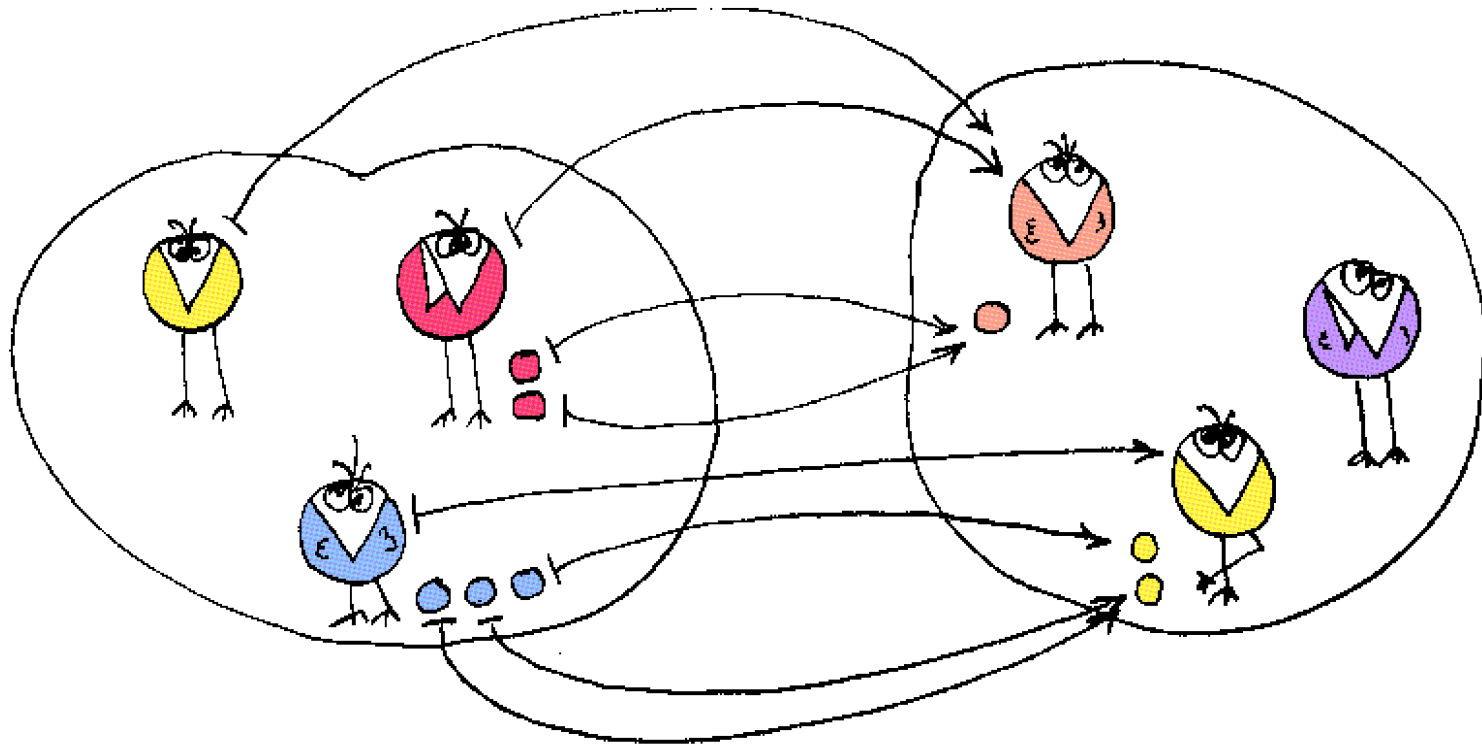
Véritable Ensemble de Shadoks



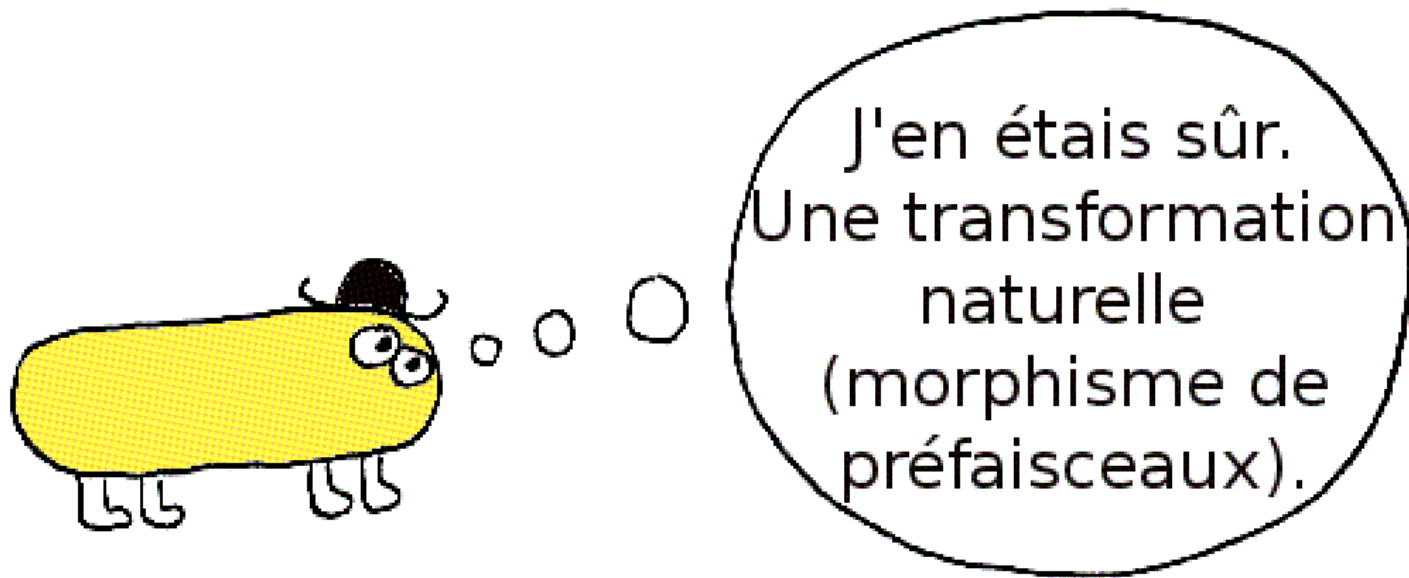
Le point de vue de l'expert



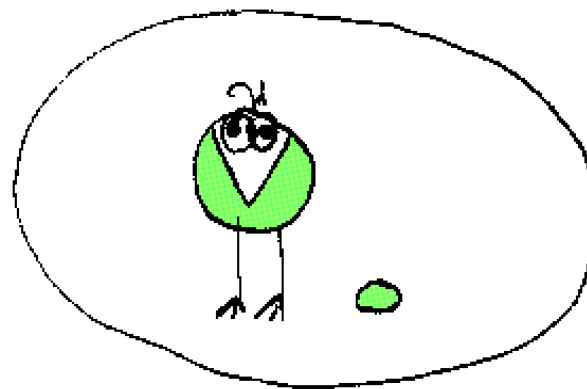
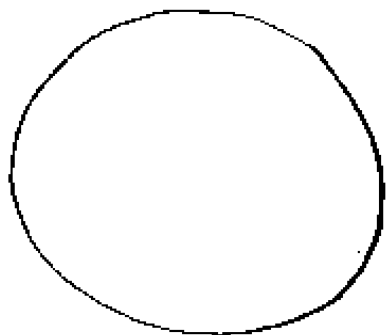
"Application"



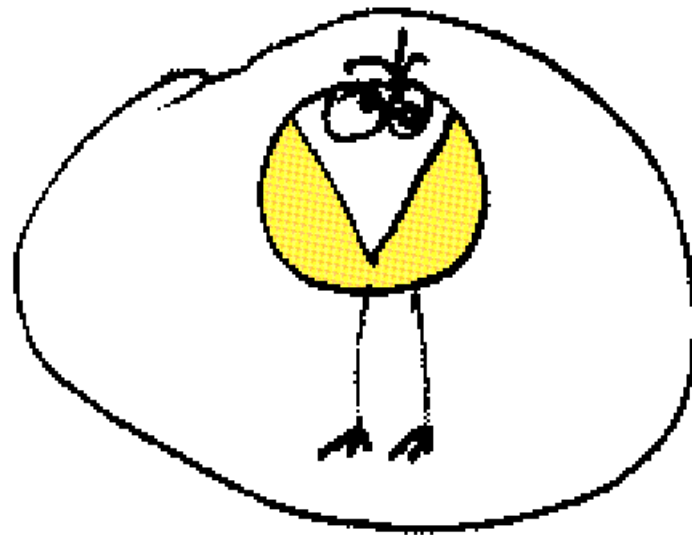
Le point de vue de l'expert



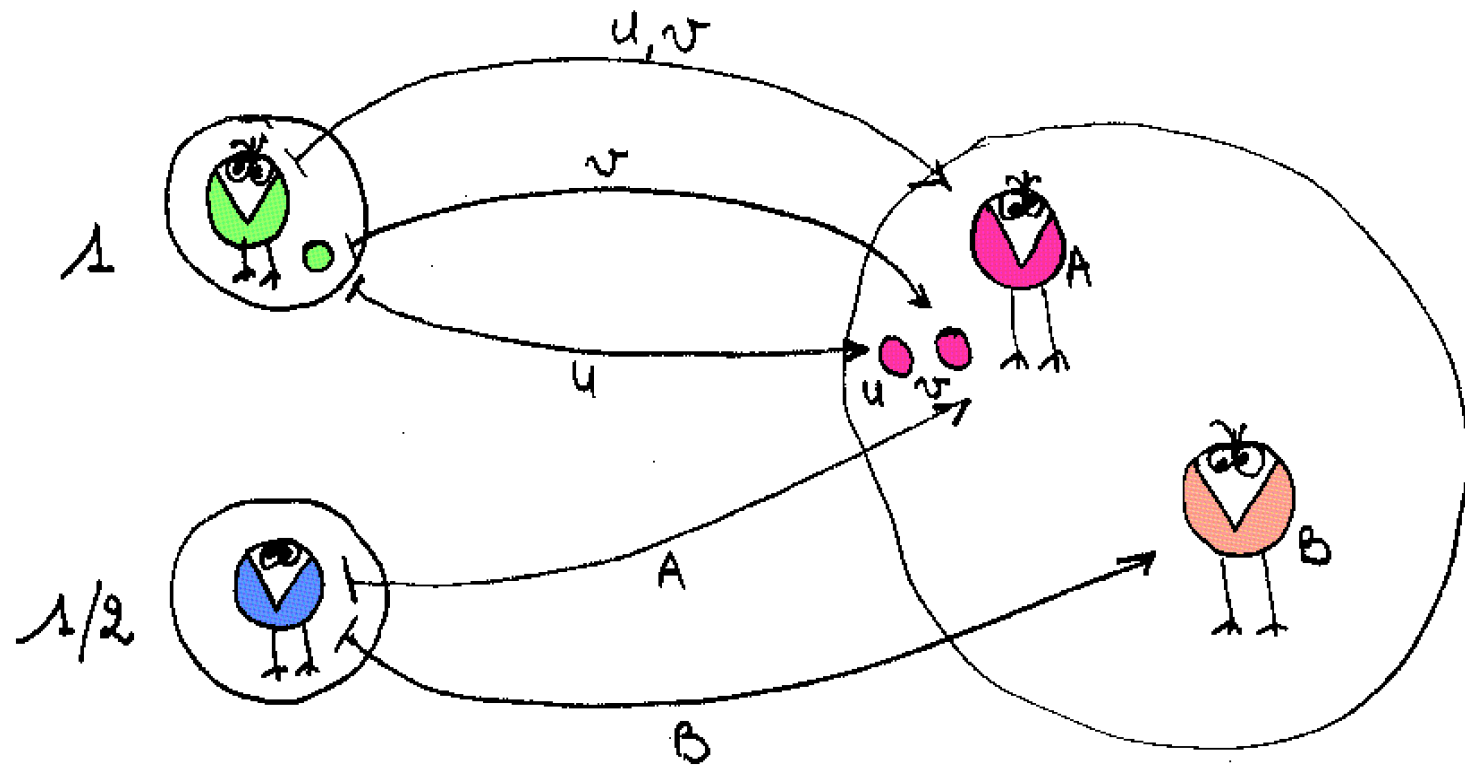
Vide et Singleton (0 et 1)



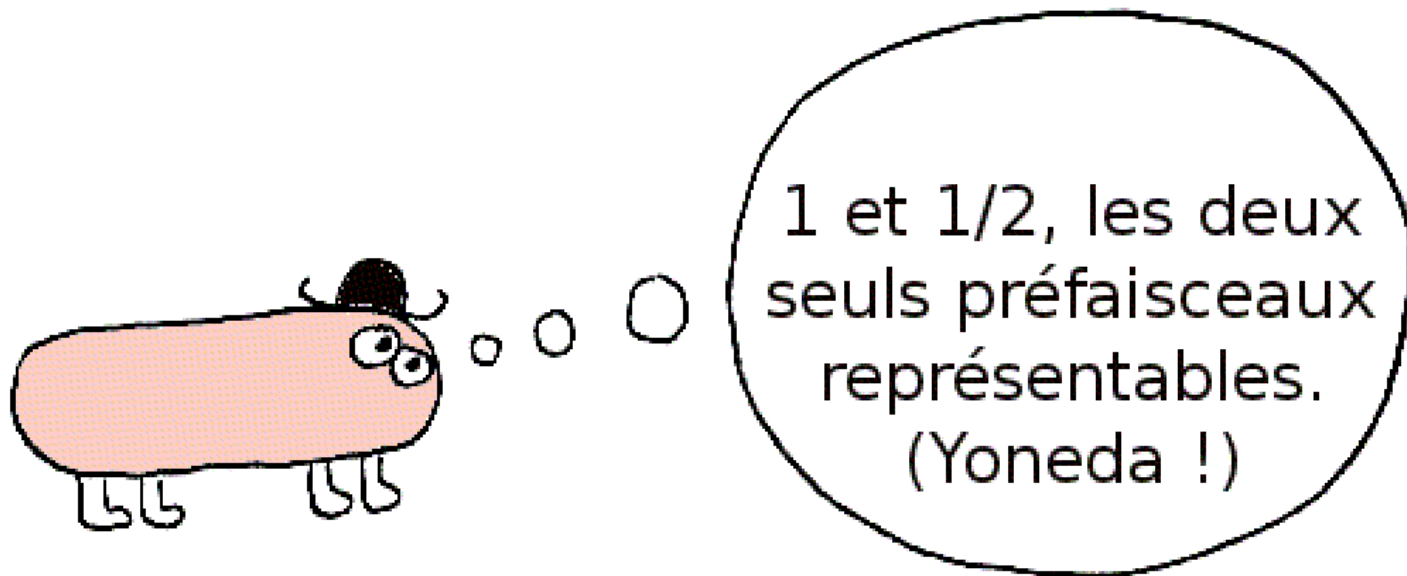
1/2



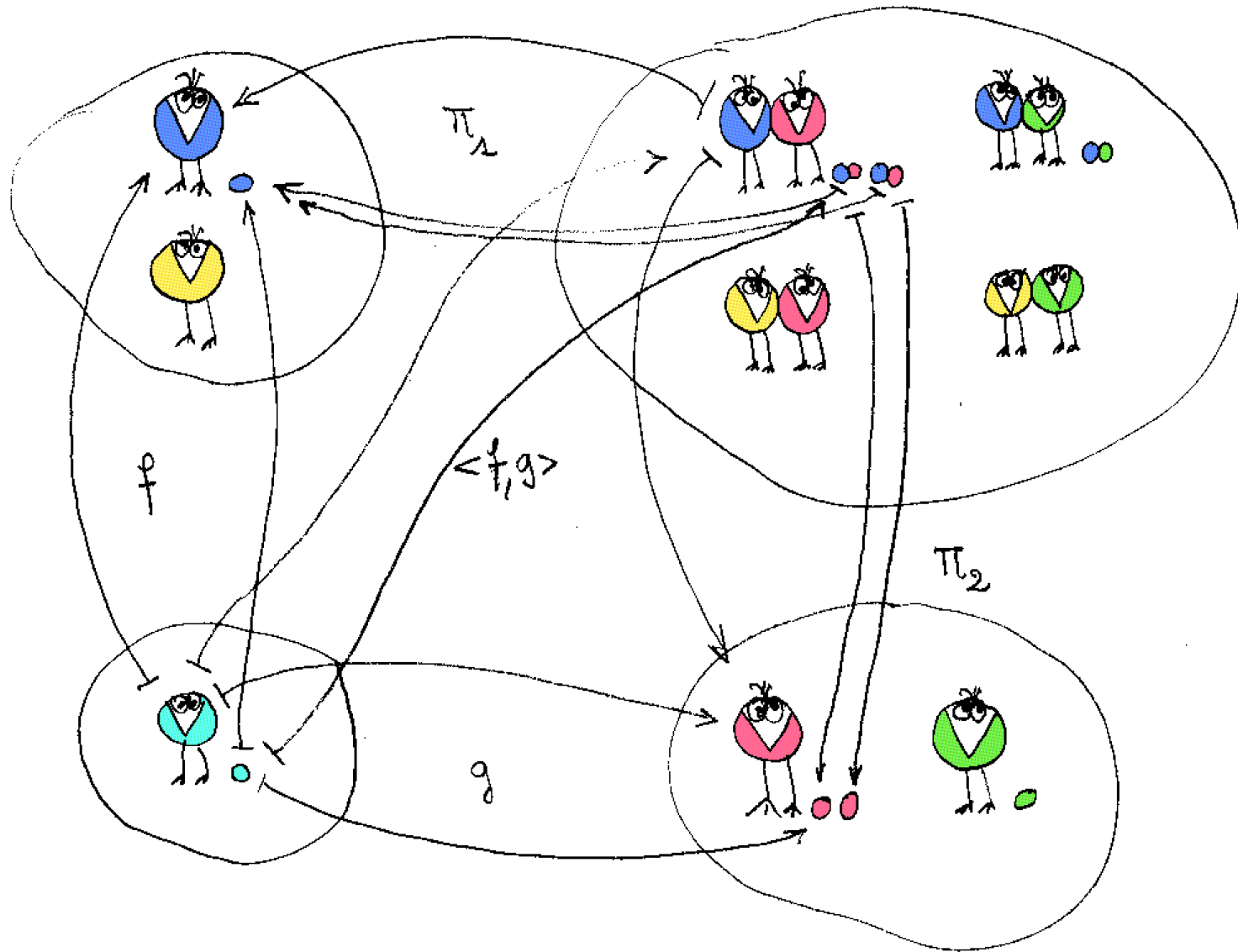
Deux ensembles importants



Le lemme de Yoneda



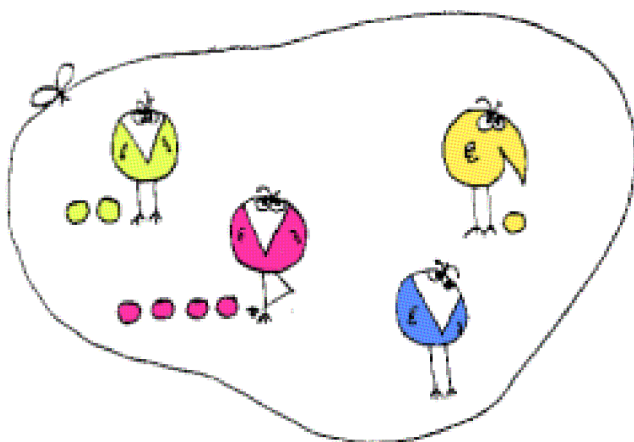
Produit "cartésien"



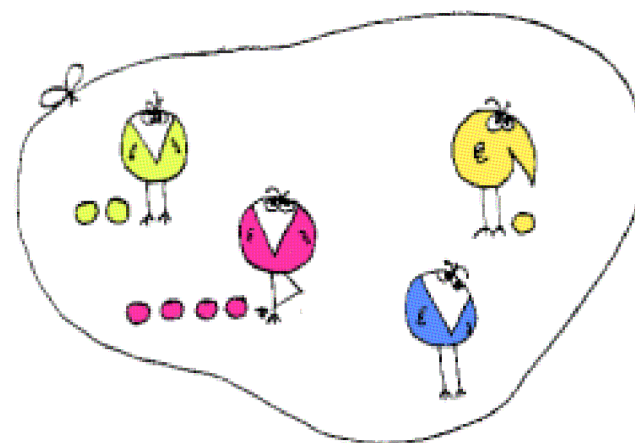
De même: limites, colimites, 0, 1, monomorphismes, épimorphismes.

Produit par 1

1 x

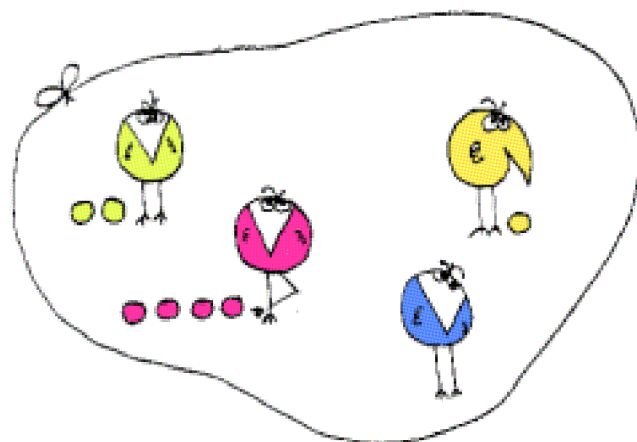


\approx

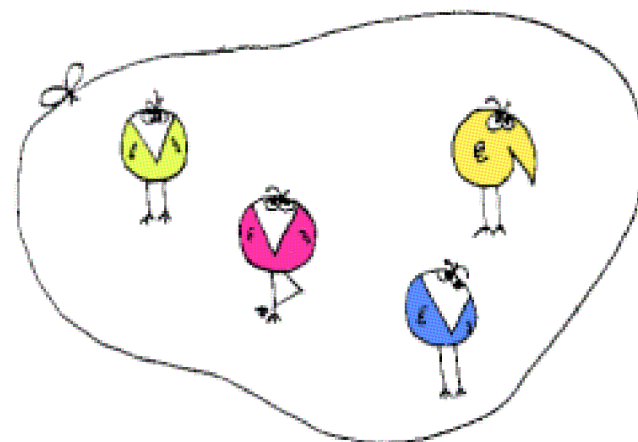


Produit par 1/2

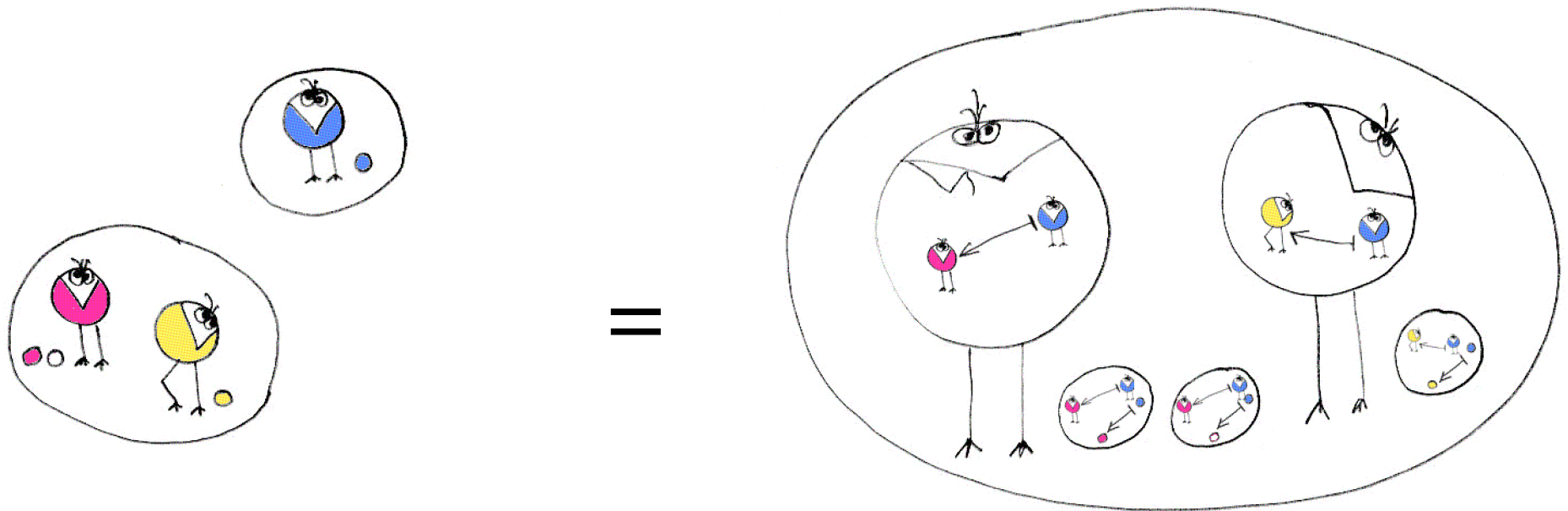
$1/2 \times$



\approx



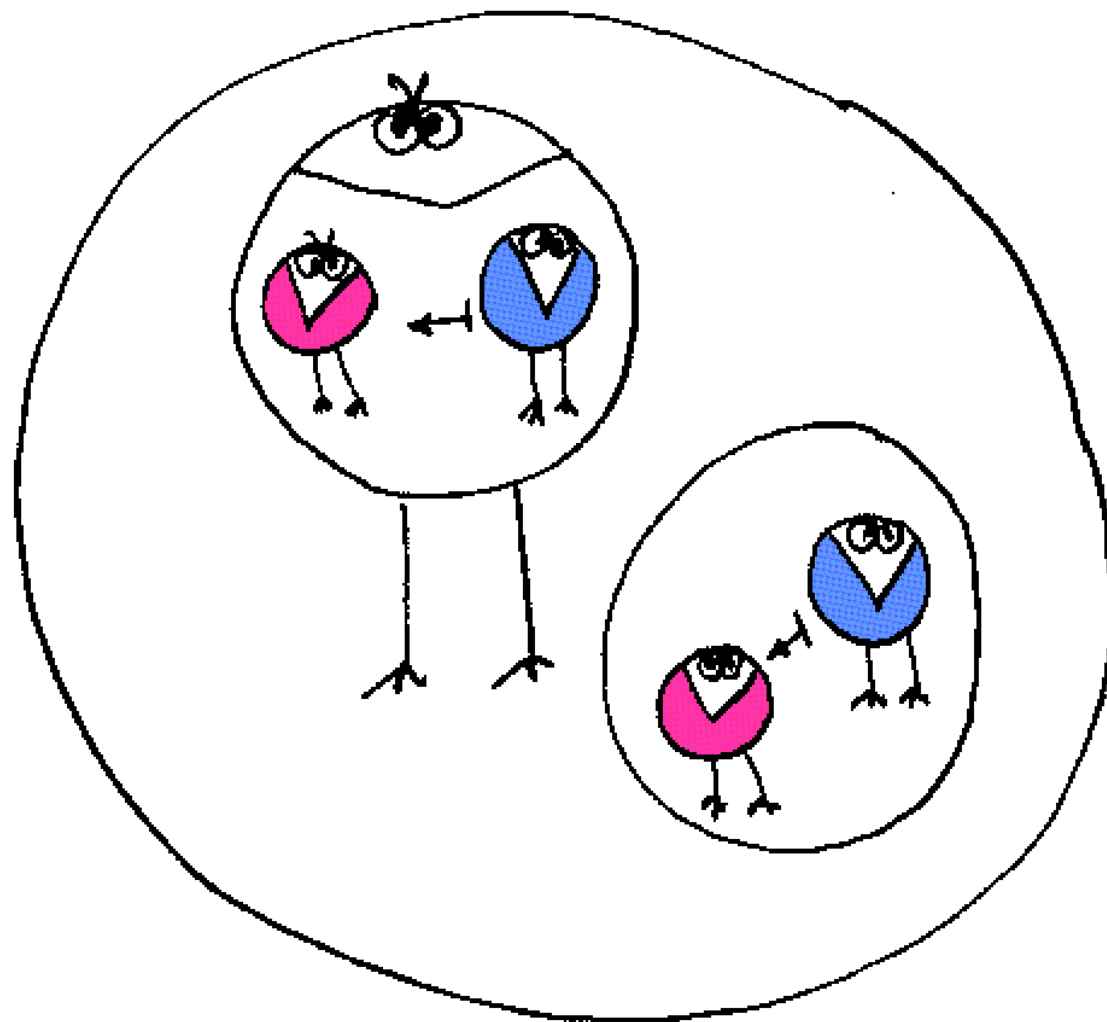
Ensemble fonctionnel



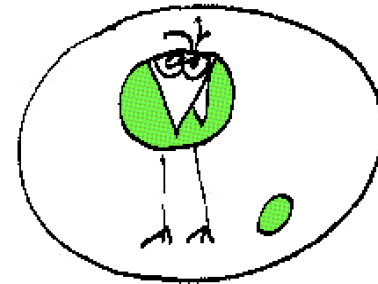
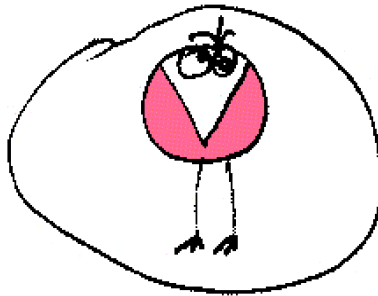
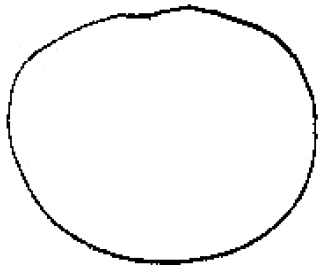
$$\frac{1 \times X \longrightarrow Y}{1 \longrightarrow Y^X}$$

$$\frac{1/2 \times X \longrightarrow Y}{1/2 \longrightarrow Y^X}$$

Qui suis-je ?

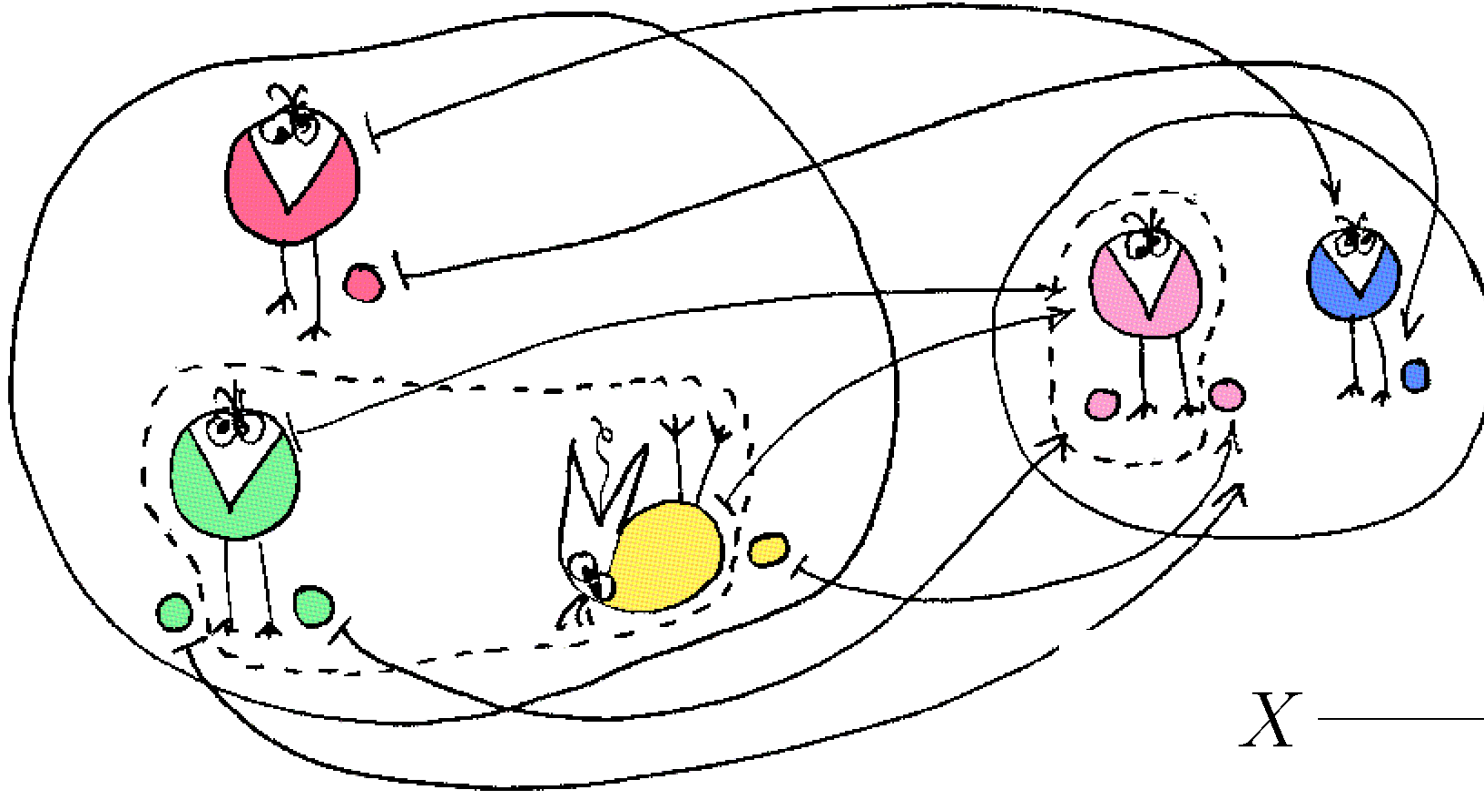


Les 3 sous-ensembles de 1



Les 3 éléments de **Sub(1)**

Image réciproque

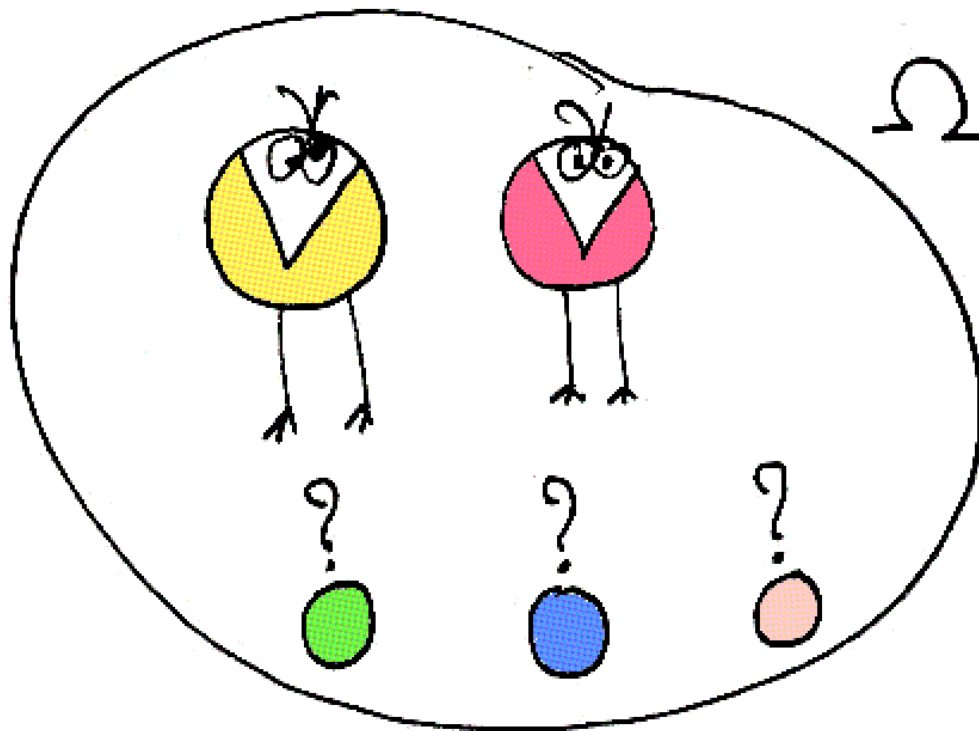


$$X \xrightarrow{f} Y$$

Sub est un foncteur

$$\mathbf{Sub}(X) \xleftarrow{\mathbf{Sub}(f)} \mathbf{Sub}(Y)$$

Valeurs de vérité ?

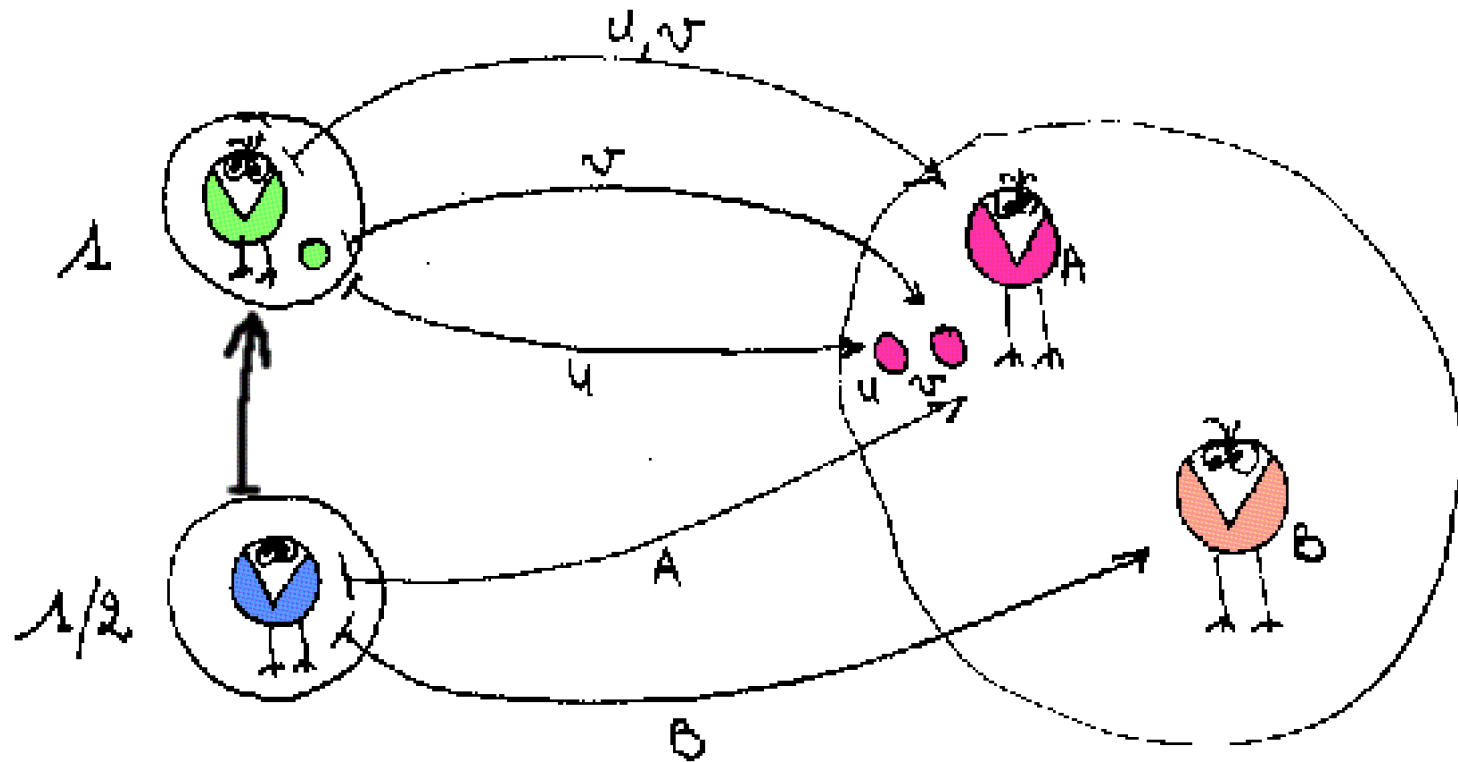


$$\mathcal{T}(X, \Omega) \xrightarrow{\cong} \text{Sub}(X)$$

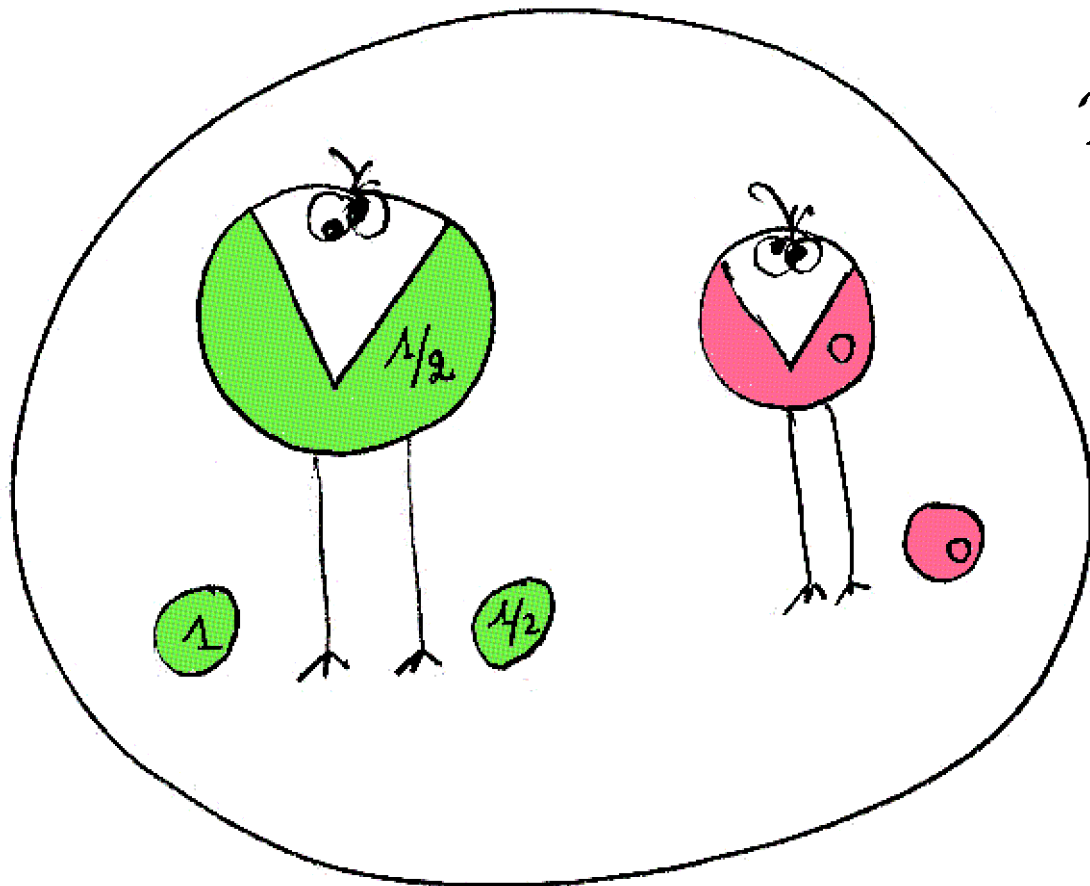
2 sous-ensembles de $\frac{1}{2}$ \Rightarrow 2 flèches de $\frac{1}{2}$ vers Ω \Rightarrow 2 shadoks dans Ω

3 sous-ensembles de 1 \Rightarrow 3 flèches de 1 vers Ω \Rightarrow 3 oeufs dans Ω

A qui est cet œuf ?

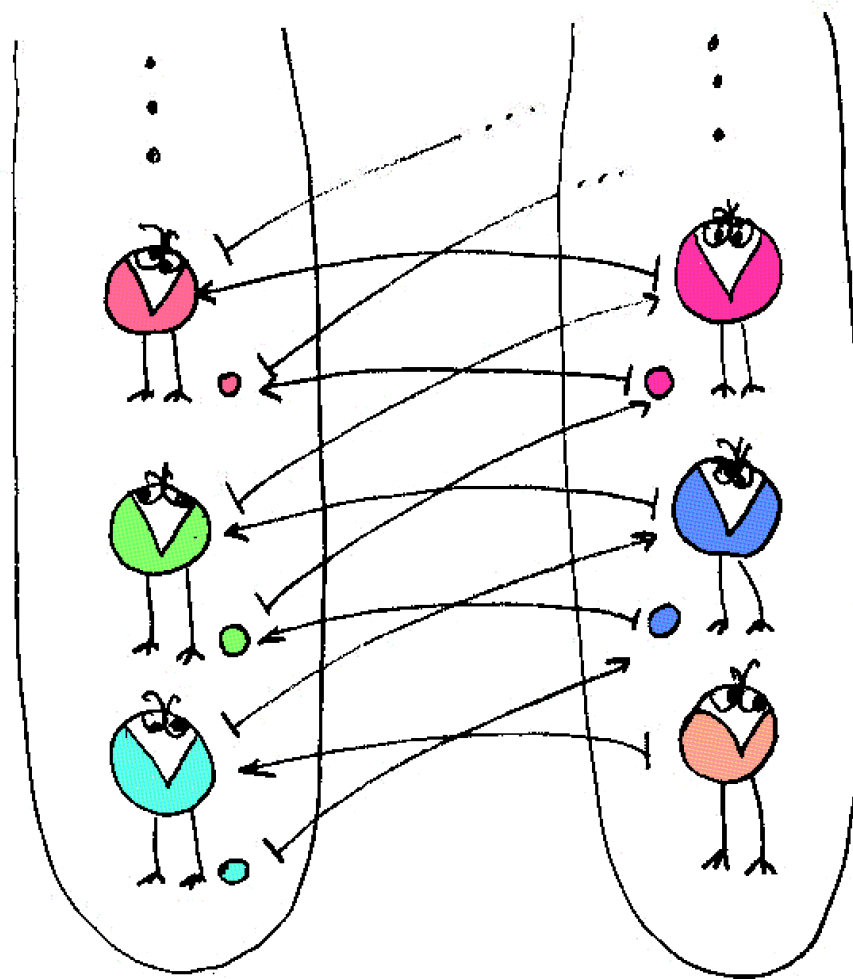


Ensemble des valeurs de vérité

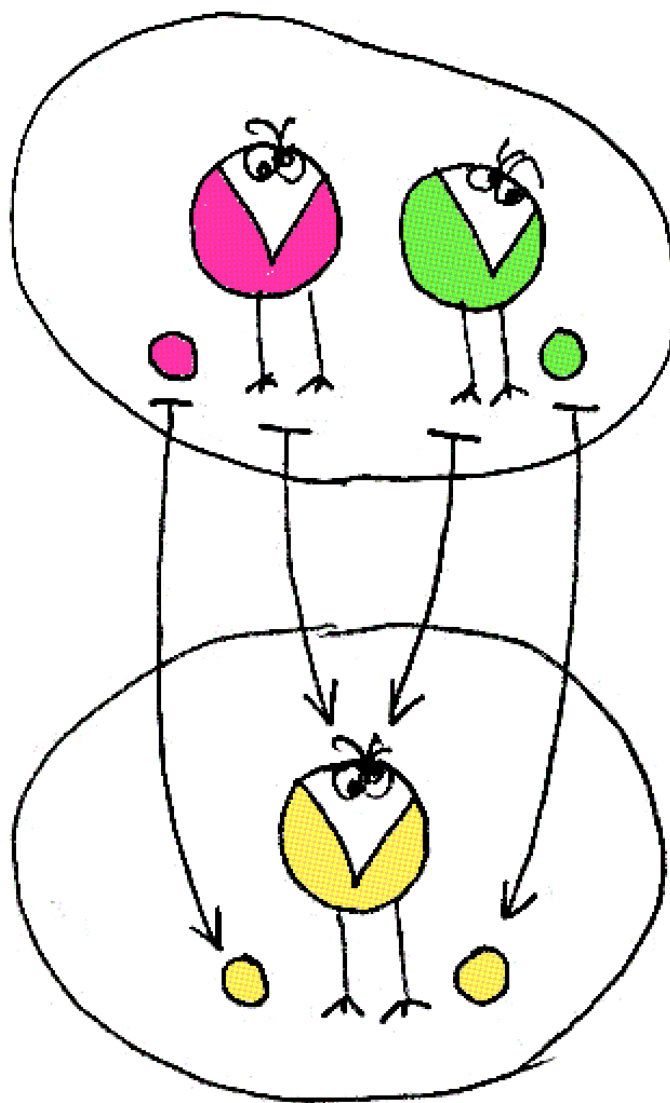


$$\begin{array}{ccc} \mathcal{T}(1, \Omega) & \xrightarrow{\cong} & \mathbf{Sub}(1) \\ \downarrow i^* & & \downarrow \mathbf{Sub}(i) \\ \mathcal{T}(\frac{1}{2}, \Omega) & \xrightarrow{\cong} & \mathbf{Sub}(\frac{1}{2}) \end{array}$$

Cantor-Bernstein



Axiome du choix



Merci de votre attention



Les Shadoks © sont une création de Jacques Rouxel